Multiplying Fractions and Whole Numbers

Essential Question What does it mean when a whole number is multiplied by a fraction? Will the product be *greater than* or *less than* the whole number?

1 **EXAMPLE: Multiplying a Fraction and a Whole Number**

You have 3 gallons of paint. You use $\frac{3}{4}$ of the paint. How many gallons did you use?

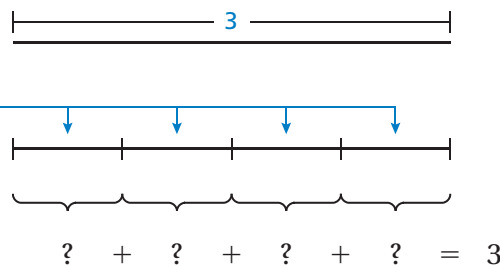**THINK ABOUT THE QUESTION:** One way to think about this question is to rewrite the question.

Words: What is $\frac{3}{4}$ of 3?   **Numbers:** $\frac{3}{4} \times 3 = ?$

Here is one way to get the answer.

● **Draw** a length of 3.

Because you want to find $\frac{3}{4}$ of the length, divide it into 4 equal sections.

$? \; + \; ? \; + \; ? \; + \; ? \; = \; 3$

Now, you need to think of a way to divide 3 into 4 equal parts.

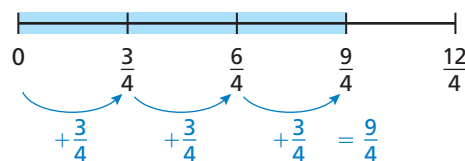● **Rewrite** the number 3 as a fraction whose numerator is divisible by 4.

Because the length is divided into 4 equal sections, multiply the numerator and denominator by 4.

$$\frac{3}{1} = \frac{3 \times 4}{1 \times 4} = \frac{12}{4}$$

In this form, you see that $\frac{12}{4}$ can be divided into four equal parts of $\frac{3}{4}$.

● Each part is $\frac{3}{4}$ gallon and you used three of them. Written as multiplication, you have
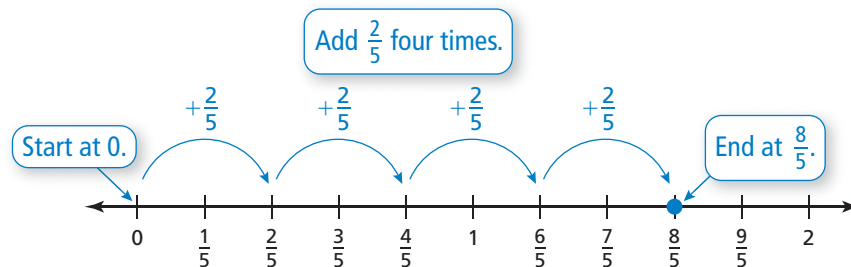
$$+\frac{3}{4} \quad +\frac{3}{4} \quad +\frac{3}{4} = \frac{9}{4}$$

$$\frac{3}{4} \times 3 = \frac{9}{4}.$$

So, you used $\frac{9}{4}$ gallons of paint.

EXAMPLE: Multiplying a Whole Number and a Fraction

Find $4 \times \dfrac{2}{5}$.

Add $\dfrac{2}{5}$ four times.

$+\dfrac{2}{5}$ $+\dfrac{2}{5}$ $+\dfrac{2}{5}$ $+\dfrac{2}{5}$

Start at 0.

End at $\dfrac{8}{5}$.

$$0 \quad \dfrac{1}{5} \quad \dfrac{2}{5} \quad \dfrac{3}{5} \quad \dfrac{4}{5} \quad 1 \quad \dfrac{6}{5} \quad \dfrac{7}{5} \quad \dfrac{8}{5} \quad \dfrac{9}{5} \quad 2$$

So, $4 \times \dfrac{2}{5} = \dfrac{8}{5}$, or $1\dfrac{3}{5}$.

Inductive Reasoning

Work with a partner. Complete the table using a number line.

Exercise	Repeated Addition
3. $\dfrac{3}{4} \times 3$	$\dfrac{3}{4} + \dfrac{3}{4} + \dfrac{3}{4} = \dfrac{9}{4}$
4. $4 \times \dfrac{2}{5}$	$\dfrac{2}{5} + \dfrac{2}{5} + \dfrac{2}{5} + \dfrac{2}{5} = \dfrac{8}{5}$
5. $\dfrac{7}{6} \times 5$	
6. $3 \times \dfrac{9}{5}$	
7. $\dfrac{1}{3} \times 12$	

What Is Your Answer?

8. a. Write a real-life problem that is related to the product $\dfrac{2}{3} \times 5$.

   b. Write a different real-life problem that is related to the product $5 \times \dfrac{2}{3}$.

   c. Are the two products equal? How is your answer related to the Commutative Property of Multiplication?

9. IN YOUR OWN WORDS What does it mean when a whole number is multiplied by a fraction? Will the product be *greater than* or *less than* the whole number?

10. Write a general rule for multiplying fractions and whole numbers.

Practice

Use what you learned about multiplying fractions and whole numbers to complete Exercises 4–11 on page 54.

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🔑 Key Idea

Multiplying Fractions and Whole Numbers

Words Multiply the numerator of the fraction by the whole number. Then write the product over the denominator.

Numbers $2 \times \dfrac{4}{9} = \dfrac{2 \times 4}{9} = \dfrac{8}{9}$

Algebra $a \cdot \dfrac{b}{c} = \dfrac{a \cdot b}{c}$, where $c \neq 0$

EXAMPLE 1 Multiplying a Whole Number and a Fraction

Find $3 \times \dfrac{7}{8}$. **Estimate** $3 \times 1 = 3$

$3 \times \dfrac{7}{8} = \dfrac{3 \times 7}{8}$ — Multiply the numerator and whole number.

— Write the product over the denominator.

$= \dfrac{21}{8}$, or $2\dfrac{5}{8}$ Simplify.

∴ So, the product is $2\dfrac{5}{8}$. **Reasonable?** $2\dfrac{5}{8} \approx 3$ ✓

EXAMPLE 2 Multiplying a Fraction and a Whole Number

Find $\dfrac{11}{12} \times 6$. **Estimate** $1 \times 6 = 6$

Remember

A **common factor** is a factor that is shared by two or more whole numbers. For example, 3 and 9 share a common factor of 3.

$\dfrac{11}{12} \times 6 = \dfrac{11 \times 6}{12}$ — Multiply the numerator and whole number.

— Write the product over the denominator.

$= \dfrac{11 \times \overset{1}{\cancel{6}}}{\underset{2}{\cancel{12}}}$ Divide out the common factor, 6, from 6 and 12.

$= \dfrac{11}{2}$, or $5\dfrac{1}{2}$ Simplify.

∴ So, the product is $5\dfrac{1}{2}$. **Reasonable?** $5\dfrac{1}{2} \approx 6$ ✓

🔴 On Your Own

Now You're Ready
Exercises 4–19

Multiply. Write the answer in simplest form.

1. $4 \times \dfrac{1}{5}$ **2.** $\dfrac{3}{8} \times 7$ **3.** $9 \times \dfrac{2}{3}$ **4.** $\dfrac{7}{10} \times 5$

EXAMPLE 3   **Standardized Test Practice**

What is the value of $\dfrac{1}{3} + x \cdot y$ when $x = 3$ and $y = \dfrac{2}{9}$?

   Ⓐ $\dfrac{2}{3}$ Ⓑ $\dfrac{20}{27}$ Ⓒ 1   Ⓓ $1\dfrac{2}{3}$

> **Remember**
>
> Be sure to use the order of operations when evaluating numerical expressions.

$$\dfrac{1}{3} + x \cdot y = \dfrac{1}{3} + 3 \cdot \dfrac{2}{9} \qquad \text{Substitute 3 for } x \text{ and } \dfrac{2}{9} \text{ for } y.$$

$$= \dfrac{1}{3} + \dfrac{3 \cdot 2}{9} \qquad \text{Multiply 3 and } \dfrac{2}{9}.$$

$$= \dfrac{1}{3} + \dfrac{\overset{1}{\cancel{3}} \cdot 2}{\underset{3}{\cancel{9}}} \qquad \text{Divide out the common factor 3.}$$

$$= \dfrac{1}{3} + \dfrac{2}{3} \qquad \text{Simplify.}$$

$$= 1 \qquad \text{Add.}$$

∴ The correct answer is Ⓒ.

EXAMPLE 4   **Real-Life Application**

About $\dfrac{9}{10}$ of the weight of a watermelon is water. How many pounds of water are in the watermelon?

To find $\dfrac{9}{10}$ of 20, multiply.   **Estimate** $\mathbf{1 \times 20 = 20}$
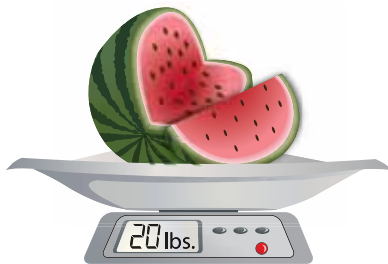
$$\dfrac{9}{10} \times 20 = \dfrac{9 \times 20}{10}$$

$$= \dfrac{9 \times \overset{2}{\cancel{20}}}{\underset{1}{\cancel{10}}} \qquad \boxed{\text{Divide out the common factor 10.}}$$

$$= 18$$

∴ So, 18 pounds of water are in the watermelon.

Reasonable? $\mathbf{18 \approx 20}$ ✓

On Your Own

Now You're Ready
Exercises 32–39

5. Find the value of $a \cdot b - \dfrac{1}{2}$ when $a = \dfrac{5}{8}$ and $b = 4$.

6. **WHAT IF?** In Example 4, a watermelon weighs 15 pounds. How many pounds of water are in the watermelon?

✓ Vocabulary and Concept Check

1. **WRITING** Describe how to multiply a fraction by a whole number.

2. **NUMBER SENSE** Use repeated addition to find the product $6 \times \dfrac{7}{8}$.

3. **NUMBER SENSE** Without multiplying, which is greater, $\dfrac{1}{3} \times 24$ or $\dfrac{1}{4} \times 24$? Explain.

Practice and Problem Solving

Multiply. Write the answer in simplest form.

① ②

4. $3 \times \dfrac{1}{7}$

5. $\dfrac{1}{8} \times 5$

6. $\dfrac{3}{4} \times 7$

7. $4 \times \dfrac{4}{9}$

8. $\dfrac{6}{5} \times 8$

9. $9 \times \dfrac{5}{3}$

10. $\dfrac{5}{6} \times 8$

11. $25 \times \dfrac{7}{10}$

12. $\dfrac{2}{3} \times 5$

13. $7 \times \dfrac{3}{10}$

14. $15 \times \dfrac{2}{3}$

15. $\dfrac{13}{14} \times 28$

16. $\dfrac{3}{20} \times 5$

17. $\dfrac{3}{8} \times 24$

18. $9 \times \dfrac{13}{24}$

19. $18 \times \dfrac{3}{4}$

ERROR ANALYSIS Describe and correct the error in finding the product.

20.

✗ $\dfrac{2}{5} \times 8 = \dfrac{2+8}{5}$

$= \dfrac{10}{5} = 2$

21.

✗ $9 \times \dfrac{3}{7} = \dfrac{3}{63}$

$= \dfrac{1}{21}$

22. **CDs** Your friend has 12 CDs and $\dfrac{3}{4}$ of them are pop music. How many CDs are pop music?

23. **OATMEAL MUFFINS** A batch of oatmeal muffins calls for $\dfrac{2}{3}$ cup of oats. How many cups of oats do you need to make four batches of muffins?

24. **RAIN** There are 365 days in a year and rain falls on $\dfrac{2}{5}$ of the days. How many days does it rain during the year?

Use compatible numbers to estimate the product.

25. $\dfrac{1}{3} \times 17$

26. $35 \times \dfrac{1}{8}$

27. $22 \times \dfrac{5}{6}$

28. $\dfrac{4}{5} \times 7$

Tell how the Commutative and Associative Properties of Multiplication can help find the product mentally. Then find the product.

29. $25 \times 6 \times \dfrac{2}{5}$

30. $\left(8 \times \dfrac{5}{9}\right) \times 18$

31. $\dfrac{3}{7} \times 13 \times 14$

ALGEBRA Evaluate the expression when $x = 6$, $y = \dfrac{3}{16}$, and $z = 20$.

③ **32.** $\dfrac{3}{4} \cdot x$

33. $8 \cdot y$
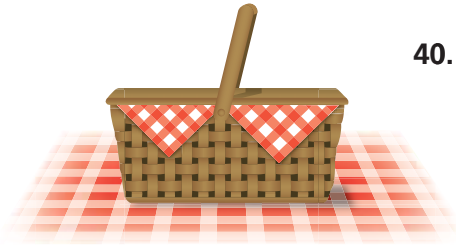
34. $\dfrac{7}{10} \cdot z$

35. $x \cdot \dfrac{4}{9}$

36. $\dfrac{2}{5}xz$

37. xyz

38. $\dfrac{3}{8} + xy$

39. $\dfrac{5}{12} + yz$

40. AREA A rectangular picnic shelter is 75 feet long by 60 feet wide. A model of the shelter is $\dfrac{1}{15}$ as wide and $\dfrac{1}{15}$ as long as the actual shelter. What is the area, in square feet, of the model of the shelter?

41. REASONING You spend $\dfrac{1}{3}$ of your money and your friend spends $\dfrac{1}{2}$ of her money shopping. Is it possible that you spend more money than your friend? Explain your reasoning.

42. NECKLACES You make bead necklaces using the beads shown in the table. Each necklace has a clasp that is $\dfrac{9}{10}$ centimeter long.

Bead		Length
Bugle		$\dfrac{9}{20}$ cm
Crow		$\dfrac{9}{10}$ cm
Tube		$\dfrac{2}{5}$ cm

a. How long is a necklace with 48 bugle beads and 24 tube beads?

b. How long is a necklace with 18 bugle beads, 18 crow beads, and 18 tube beads?

c. **Reasoning** You want to make a necklace 38 centimeters or longer. You have 26 bugle beads, 18 crow beads, and 16 tube beads. Do you have enough beads? If not, what is the smallest number of bugle beads you need to add to make the necklace? Explain how you found your answer.

Fair Game Review *What you learned in previous grades & lessons*

Evaluate the expression.

43. $\dfrac{3 \times 5}{2 \times 4}$

44. $\dfrac{9 \times 2}{1 \times 5}$

45. $\dfrac{4 \times 8}{7 \times 25}$

46. $\dfrac{10 \times 13}{3 \times 11}$

47. MULTIPLE CHOICE What is the area of a parallelogram with a base of 12 inches and a height of 4 inches?

A 24 in.

B 48 in.

C 24 in.^{2}

D 48 in.^{2}